Data Visualization (CIS/DSC 468)

Isosurfaces and Volume Rendering

Dr. David Koop
Fields & Grids

• Fields:
  - Values come from a **continuous** domain, infinitely many values
  - **Sampled** at certain positions to approximate the entire domain
  - Often measurements of natural or simulated phenomena
  - Examples: temperature, wind speed, tissue density, pressure, speed, electrical conductance

• Grids: geometry (positions) and topology (connections)
Fields & Grids

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• Grids: geometry (positions) and topology (connections)

- uniform
- rectilinear
- structured
- unstructured

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Fields in Visualization

Scalar Fields
(Order-0 Tensor Fields)

Vector Fields
(Order-1 Tensor Fields)

Tensor Fields
(Order-2+)

Each point in space has an associated...

Scalar

Vector

Tensor

\[
\begin{bmatrix}
 s_0 \\
 v_0 \\
 v_1 \\
 v_2 
\end{bmatrix}
\]

\[
\begin{bmatrix}
 \sigma_{00} & \sigma_{01} & \sigma_{02} \\
 \sigma_{10} & \sigma_{11} & \sigma_{12} \\
 \sigma_{20} & \sigma_{21} & \sigma_{22} 
\end{bmatrix}
\]
What are different types of interpolation?
Nearest Neighbor Interpolation

Value at 2.2?
Linear Interpolation

Value at 2.2?
Visualizing Volume (3D) Data

- 2D visualization slice images (or multi-planar reformating MPR)
- *Indirect* 3D visualization isosurfaces (or surface-shaded display SSD)
- *Direct* 3D visualization (direct volume rendering DVR)

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Visualizing Volume (3D) Data

(a) 2D slice

(b) Volume Rendering

[J. Kniss, 2002]
Visualizing Volume (3D) Data

(a) An isosurfaced tooth.

(b) Multiple isosurfaces.

[J. Kniss, 2002]
Assignment 4

- Changing value + reordering interaction
- Brushing (linked highlighting)
How have we encoded 3D data before? Hint: Think about maps
Isolines (2D)

- Isoline: a line that has the same scalar value at all locations
- Example: Topographical Map

[USGS via Wikipedia]
How?

• Given an isovalue, we want to draw the isocontours corresponding to that value
• Remember we only have values defined at grid points
• How do we get isolines or isosurfaces from that data?
• Can we use the ideas from interpolation?
Generating Isolines

![Isolines grid](image)

**Figure 2.4.**

(a) 2D scalar grid. (b) Black vertices are positive. Vertex \( v \) with scalar value \( s_v \) is positive if \( s_v > 0 \) and negative if \( s_v < 0 \). Note that \( s_v = 0 \) for one grid vertex. (c) Isocontour with vertices at edge midpoints (before linear interpolation). (d) Isocontour with isovalue 5.

The isocontour lookup table, Table \([\kappa]\), contains six entries, one for each configuration. Each entry, Table \([\kappa] \), is a list of the \( +/− \kappa \) pairs.

In Figure 2.3 the isocontour edges are drawn connecting the midpoints of each square edge. This is for illustration purposes only. The geometric locations of the isocontour vertices are not defined by the lookup table.

The isocontour lookup table is constructed on the unit square with vertices \((0, 0), (1, 0), (0, 1), (1, 1)\). To construct the isocontour in grid square \((i, j)\), we have to map pairs of unit square edges to pairs of square \((i, j)\) edges. Each vertex \( v = (v_x, v_y) \) of the unit square maps to \( v + (i, j) = (v_x + i, v_y + j) \). Each edge \( e \) of the unit square with endpoints \((v, v')\) maps to edge \( e + (i, j) = (v + (i, j), v' + (i, j)) \). Finally, each edge pair \((e_1, e_2)\) maps to \((e_1 + (i, j), e_2 + (i, j))\).

The endpoints of the isocontour edges are the isocontour vertices. To map each isocontour edge to a geometric line segment, we use linear interpolation.

[R. Wenger, 2013]
Generating Isolines (Isovalue=5)

![Diagram showing scalar grid and isocontour]

(a) Scalar grid. (b) The +/− grid.
(c) Midpoint vertices. (d) Isocontour.

Figure 2.4.

(a) 2D scalar grid. (b) Black vertices are positive. Vertex \( v \) with scalar value \( s_v \) is positive if \( s_v > 5 \) and negative if \( s_v < 5 \). Note that \( s_v = 5 \) for one grid vertex. (c) Isocontour with vertices at edge midpoints (before linear interpolation). (d) Isocontour with isovalue 5.

The isocontour lookup table, Table, contains sixteen entries, one for each configuration. Each entry, Table\([\kappa]\), is a list of the \( E^+/-\kappa \) pairs.

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[R. Wenger, 2013]
Generating Isolines w/ Interpolation (Isovalue=5)

[Figure 2.3]

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Figure 2.4. (a) 2D scalar grid. (b) Black vertices are positive. Vertex \( v \) with scalar value \( s_v \) is positive if \( s_v > 5 \) and negative if \( s_v < 5 \). Note that \( s_v = 5 \) for one grid vertex. (c) Isocontour with vertices at edge midpoints (before linear interpolation). (d) Isocontour with isovalue 5.

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The endpoints of the isocontour edges are the isocontour vertices. To map each isocontour edge to a geometric line segment, we use linear interpolation to generate the isolines.
Marching Squares

Figure 2.10. Red, positive regions and blue, negative regions for each square configuration. The green isocontour is part of the positive region. Black vertices are positive.

Proof of Properties 1 & 2:
The Marching Squares isocontour consists of a finite set of line segments, so it is piecewise linear. These line segments intersect only at their endpoints and thus form a triangulation of the isocontour. The endpoints of these line segments lie on the grid edges, confirming Property 2. □

Property 3.
The isocontour intersects every bipolar grid edge at exactly one point.

Property 4.
The isocontour does not intersect any negative or strictly positive grid edges.

Proof of Properties 3 & 4:
Each isocontour edge is contained in a grid square. Since the grid squares are convex, only isocontour edges with endpoints (vertices) on the grid edge intersect the grid edge. If the grid edge has one positive and one negative endpoint, the unique location of the isocontour vertex on the grid edge is determined by linear interpolation. Thus the isocontour intersects a bipolar grid edge at only one point.

If the grid edge is negative or strictly positive, then no isocontour vertex lies on the grid edge. Thus the isocontour does not intersect negative or strictly positive grid edges. □

Within each grid square the isocontour partitions the grid square into two regions. Let the positive region for a grid square $c$ be the set of points which can be reached by a path $\zeta$ from a positive vertex. More precisely, a point $p$ is in the positive region of $c$ if there is some path $\zeta \subset c$ connecting $p$ to a positive vertex of $c$ such that the interior of $\zeta$ does not intersect the isocontour. A point $p$ is in the negative region of $c$ if there is some path $\zeta \subset c$ connecting $p$ to a negative vertex of $c$ such that $\zeta$ does not intersect the isocontour. Since any path $\zeta \subset c$ from a positive to a negative vertex must intersect the isocontour, the positive and negative regions form a partition of the square $c$.

[R. Wenger, 2013]
Ambiguous Configurations

- There are some cases for which we cannot tell which way to draw the isolines...

![Diagram showing ambiguous configurations](image-url)

[Figure 2.12. Ambiguous square configurations.]

[Figure 2.13. Topologically distinct isocontours created by using different isocontours for the ambiguous configuration in the central grid square.]

While the choice of isocontours for the ambiguous configurations changes the isocontour topology, any of the choices will produce isocontours that are 1-manifolds and strictly separate strictly positive vertices from negative vertices. As we shall see, this is not true in three dimensions.

2.3 Marching Cubes

2.3.1 Algorithm

The three-dimensional Marching Cubes algorithm follows precisely the steps in the two-dimensional Marching Squares algorithm. Input to the Marching Cubes algorithm is a scalar grid with two topologically distinct isocontours created by different resolutions of the ambiguous configurations. The first isocontour has two components while the second has one.

[R. Wenger, 2013]
Ambiguous Configurations

- Either works for marching squares, this isn't the case for 3D

Figure 2.12. Ambiguous square configurations.

Figure 2.13. Topologically distinct isocontours created by using different isocontours for the ambiguous configuration in the central grid square.

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As we shall see, this is not true in three dimensions.

2.3 Marching Cubes

2.3.1 Algorithm

The three-dimensional Marching Cubes algorithm follows precisely the steps in the two-dimensional Marching Squares algorithm. Input to the Marching Cubes algorithm consists of an isosurface in a scalar grid with two topologically distinct isocontours created by different resolutions of the ambiguous configurations. The first isocontour has two components while the second has one.

[R. Wenger, 2013]
3D: Marching Cubes

- Same idea, more cases [Lorensen and Cline, 1987]

![Diagram of isosurfaces for 22 distinct cube configurations](image)

[R. Wenger, 2013]
Incompatible Choices

• If we have ambiguous cases where we choose differently for each cell, the surfaces will not match up correctly—there are holes.
• Fix with the asymptotic decider [Nielson and Hamann, 1991].

Example: case 3c, both versions are plausible.
Marching Cubes Algorithm

• For each cell:
  - Classify each vertex as inside or outside (≥, <) — 0 or 1
  - Take the eight vertex classifications as a bit string
  - Use the bit string as a lookup into a table to get edges
  - Interpolate to get actual edge locations
  - Compute gradients
  - Resolve ambiguities

• Render a bunch of triangles: easy for graphics cards
Multiple Isosurfaces

- Topographical maps have multiple isolines to show elevation trends
- Problem in 3D? **Occlusion**
- Solution? Transparent surfaces
- Issues:
  - Think about color in order to make each surface visible
  - Compositing: how do colors "add up" with multiple surfaces
  - How to determine good isovalue? 

[J. Kniss, 2002]
Isosurface Issues

• Must select isovalues, how to determine "good" values
• Hard boundary: any noise or uncertainty leads to jagged boundaries
Visualizing Volume (3D) Data

- **2D visualization**
  - slice images (or multi-planar reformating MPR)

- **Indirect**
  - 3D visualization isosurfaces (or surface-shaded display SSD)

- **Direct**
  - 3D visualization (direct volume rendering DVR)

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Volume Rendering vs. Isosurfacing

(a) Direct volume rendered  
(b) Isosurface rendered

[Kindlmann, 1998]