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REASONING UNDER UNCERTAINTY FOR SHILL DETECTION IN ONLINE AUCTIONS USING DEMPSTER-SHAFER THEORY*

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This paper describes the design of a decision support system for shill detection in online auctions. To assist decision making, each bidder is associated with a type of certification, namely shill, shill suspect, or trusted bidder, at the end of each auction's bidding cycle. The certification level is determined on the basis of a bidder's bidding behaviors including shilling behaviors and normal bidding behaviors, and thus fraudulent bidders can be identified. In this paper, we focus on representing knowledge about bidders from different aspects in online auctions, and reasoning on bidders' trustworthiness under uncertainties using Dempster-Shafer theory of evidence. To demonstrate the feasibility of our approach, we provide a case study using real auction data from eBay. The analysis results show that our approach can be used to detect shills effectively and efficiently. By applying Dempster-Shafer theory to combine multiple sources of evidence for shill detection, the proposed approach can significantly reduce the number of false positive results in comparison to approaches using a single source of evidence.

Keywords: Reasoning under uncertainty; knowledge representation; online auctions; shill detection; Dempster-Shafer theory.

1. Introduction

Online auction houses have become a convenient trading platform for millions of sellers and buyers. Auction websites handle a great variety of goods, from antique vases and record albums to brand new trucks and $40,000 computer servers [1]. The popularity of online auctions continues to flourish as it enables ordinary people to become instant businessmen. Online auction services are still growing, including high profile companies such as Amazon auctions, Overstock auctions, and uBid, just to name a few. However, the nature of online auctions also presents some serious problems. For example, because it is easy for individuals and businesses to establish and run online auction stores, and at

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the same time, online auction users do not deal with each other face to face, this contemporary business medium is currently facing an important challenge – auction frauds, which are serious illegal activities in online auctions.

Shill bidding is one of the most prevalent forms of auction frauds that violate the integrity of online auctions. A shill is a person who pretends to be a legitimate buyer and feigns enthusiasm for an auctioned item by bidding up the auction price. The role of a shill is typically played by an associate of the seller, i.e., the owner of the auctioned item. In some cases, it can also be played by the seller himself, who poses as a legitimate buyer under a fake online user ID. The ultimate purpose of employing shills is to trick legitimate buyers into paying more than they would if there were no auction frauds [2].

In recent years, shill bidding has started to capture people's attention. According to recent criminal charges, some felons placed shill bids in thousands of auctions, driving up the price from several dollars to a few thousand dollars [3]. Although the punishment for auction fraud could be severe (e.g., several years in prison with fines), shill bidding is still very popular due to a speculative risk for a huge potential gain. We notice that even some sellers in eBay's Power Seller program [4] with 100% positive feedback confessed in online forums that they would have made much less money if they did not use shill bidding.

However, shill bidding is very difficult to detect. Shill bidding usually occurs without leaving obvious direct physical evidence, thus it cannot be easily captured by the victims. Kauffman and Wood examined the effects of shill bidding on the final bidding price in rare coin auctions, and showed that some bidders might view shill bids as signals that an item was worth more, thus they would be likely to pay more than other bidders who could not see the signals [5].

To protect online business, shill bidding behaviors have been researched, and many shill bidding strategies or patterns have been identified in order to help investigate auction frauds. However, most of the previous findings involve uncertainties. For example, we might identify the following shill pattern in real auction data: "When a bidder tends to place bids in an auction with a higher current bidding price than the current price in a concurrent auction with an identical auctioned item, the bidder might be a shill" [6]. This is not a certain rule for shill detection because it is also possible that some experienced buyers may prefer highly rated sellers with better reputation for quality of service to lower-rated ones, even at the cost of a higher payment. Furthermore, to support automated detection of shill bidders, we need a consistent approach to representing and quantifying auction and bidder related knowledge. For instance, by calculating and analyzing a bidder's winning ratio, we may have a better idea about the bidder's actual bidding intention – if a bidder wins often, the bidder is not likely a shill because a shill typically avoid winning auctions.

To address these problems, in this paper, we propose a decision support approach to certifying bidders' behaviors immediately after each auction's bidding cycle, but before the auction is officially closed. Similar to resolving a criminal case, we first collect evidence that supports a bidder being a shill as well as evidence that supports a bidder being an honest bidder. Since each piece of evidence involves uncertainties, it is appropriate to employ some formal reasoning technique [7, 8]. In this context, we propose to use belief functions in Dempster-Shafer (D-S) theory [9, 10, 11] to model the uncertainties associated with different pieces of evidence pertaining to varied bidding properties. This allows us to explicitly represent the uncertainties and combine knowledge from different sources of evidence to produce an aggregated assessment. Based on the assessment, a certification is issued to each bidder, which can assist both auction houses and auction participants in deciding the trustworthiness of bidders. This work extends our previous proposed framework of using Dempster-Shafer theory for shill detection [13]. The major extensions are as follows. First, auction-level properties and evidence are introduced, which complements the bid-level evidence for providing a macro-examination of auctions. Second, in the previous work, the focus was only on quantifying the degree of belief concerning if a bidder is a shill, rather than considering both cases – if a bidder is a shill or is not a shill. In this paper, each piece of evidence is determined to support either shilling behavior or normal bidding behavior based on the analysis of the corresponding quantified bidding property. Algorithms for calculating the belief of being not a shill as well as belief of being a shill are presented in this extended work. Therefore, the scope of the decision support system is significantly improved.

The rest of the paper is organized as follows. Dempster-Shafer theory is reviewed in Section 2. Section 3 introduces our abstract model for shill detection as well as the certification framework. We explicitly identify bidders' general bidding properties for shilling behaviors and normal bidding behaviors in Section 4. Section 5 provides the details of the shill certification process. A case study and analysis results are presented in Section 6. Section 7 discusses related work. Section 8 concludes the paper and mentions future work.

2. Dempster-Shafer Theory of Evidence

Dempster-Shafer (D-S) theory of evidence is a mathematical theory that was developed by Dempster and Shafer in 1976 as a new approach for representing uncertainties and expressing conflict involved in a set of evidence [9, 10]. D-S theory has often been used to combine information (evidence) from different sources to calculate the probability of an event. Generally, D-S theory differs from traditional probability theory in that the former allows the explicit representation of ignorance and uncertainties in the evidence combination process. Furthermore, D-S theory allows assigning a probability to not only singletons but also a set of multiple alternative elements [11, 12]. These unique characteristics make D-S theory particularly attractive to designing and implementing complex systems. In this section we highlight some of the key concepts of D-S theory, including some examples from our domain of interest, shilling behavior in auctions.

The belief distribution of the D-S theory is based on a *universe of discourse* Θ (also called *frame of discernment*) that consists of a finite set of mutually exclusive atomic states in a problem domain [9]. For example, in the auction shill detection domain, the frame of discernment for a bidder is $\Theta=\{shill, \sim shill\}$. The power set 2^{Θ}, which is the set of all possible subsets of Θ including the empty set, can be denoted as 2^{Θ} = {Ø, {*shill*}, {~*shill*}, Θ }.

There are three important functions in D-S theory: Basic Mass Assignment function, Belief function, and Plausible function [11]. The Basic Mass Assignment (BMA) function is m: $2^{\Theta} \rightarrow [0,1]$. It assigns a belief mass in the interval between 0 and 1 to each subset of the power set. The belief mass represents the impact of a piece of evidence to the subset of 2^{Θ}. The BMA function should verify the following two equations:

$$\sum_{A \in 2^{\Theta}} m(A) = 1 \quad (1) \qquad\qquad m(\varnothing)=0 \quad (2)$$

The empty set Ø represents a contradiction, which cannot be true in any state. Therefore, the BMA for Ø is assigned 0. The basic mass assignment *m*(Θ) can be interpreted as the measurement of conflict (in our application both states of shill and ~shill are present) and a mass is computed for the conflict. For the shill detection problem, Eq. (1) and Eq. (2) imply that *m*(*shill*) +*m*(~*shill*) +*m*(Θ) = 1.

To obtain the overall belief of *A*, one must take the sum of beliefs on all subsets of *A*. As defined in Eq. (3), a *belief function* is defined as the mass sum of all *B*s, which are subsets of *A*.

$$bel(A) = \sum_{B \subset A} m(B) \tag{3}$$

D-S theory allows a belief of a subset of 2^{Θ} to be represented by intervals, bounded by belief and plausibility [12] – for example, *bel*(*{shill}*) ≤ *P*(*{shill}*) ≤ *pl*(*{shill}*). The plausibility of *A* specifies the likelihood that it is not any other subset in 2^{Θ}. The quantity of plausibility of *A* is equal to one minus *bel*(~*A*), that is *Pl*(*A*)=1-*bel*(~*A*). For example, the degree of plausibility for shill is: Pl(shill) = m({shill}) +m({shill; ~shill}). According to Eq. (3), it is easy to derive that the quantity of plausibility of *A* is equal to the sum of the masses of *B*, whose intersection with *A* is not empty, as shown in Eq. (4). For all $A \in \Theta$, *bel*(*A*) forms a lower bound for A that could possibly happen, and *pl*(*A*) forms an upper bound for A to happen, given by Eq. (5).

$$pl(A) = \sum_{B|B \cap A \neq \varnothing} m(B) \tag{4}$$

$$bel(A) \le P(A) \le pl(A) \tag{5}$$

Knowing any of the three functions *m*, *bel*, and *pl*, the other two can be deduced using Eq. (3) and Eq. (4) [9].

Given independent belief functions over the same frame of discernment, we can combine the beliefs into a common agreement concerning a subset of 2^{Θ} and quantify the conflicts using Dempster's rule of combination [9]. Given two masses $m_1$and m_2, this combination computes a *joint mass* for the two pieces of evidence under the same frame of discernment. It is calculated as follows:

$$m_{1,2}(A) = \Big(\sum_{Y_1 \cap Y_2 = A} m_1(Y_1) m_2(Y_2)\Big) / K \text{ , where } K = 1 - \sum_{Y_1 \cap Y_2 = \varnothing} m_1(Y_1) m_2(Y_2) \tag{6}$$

Note that K represents the renormalization factor, which is equal to one minus the amount of the conflicts between two masses pertaining to the subset A of the frame of discernment.

The combination rule is usually denoted as the orthogonal sum of belief values; in other words, the combination of belief from evidence a and belief from evidence b is denoted as $bel_{a,b}=bel_a \oplus bel_b$. Therefore, the global belief of A can be represented as $bel(A) = \oplus\ bel_i$, for all pieces of evidence that supports A.

To illustrate the concepts, consider a subset X of 2^{Θ} and evidence E_1 that yield a set of values represented by $m_{E1}(\{x\})$, $m_{E1}(\{\sim x\})$, and $m_{E1}(\{x,\sim x\})$. Suppose that evidence E_1 may provide, in general, some support that X is true, i.e., event x occurs, or some support that X is not true, i.e., event $\sim x$ occurs. In terms of the mass function, the BMAs for x and $\sim x$ are m_{E1} $(\{x\})$ and m_{E1} $(\{\sim x\})$, respectively. Lack of knowledge about whether x occurs or not is represented by m_{E1} $(\{x,\sim x\})$. The sum of the three values is one. i.e., $m_{E1}(\{x\}) + m_{E1}(\{\sim x\}) + m_{E1}(\{x,\sim x\}) = 1$. We can further assume that evidence E_1 is either reliable with probability 0.9 or unreliable with probability 0.1. Now, using our shilling behavior example, suppose that evidence E_1 supports that bidder i is a shill with 100% certainty. Considering E_1's reliability, E_1 gives 0.9 degree of belief for supporting that bidder i is a shill (i.e., $m_{E1}(\{x\}) = 0.9$), but zero degree of belief that bidder i is honest (i.e., $m_{E1}(\{\sim x\}) = 0$) because the evidence does not support bidder i is honest. The remaining degree of belief (0.1) is due to the uncertainty, i.e., $m_{E1}(\{x, \sim x\}) = 0.1$.

3. Shill Detection Under Uncertainty

3.1. *An abstract model*

Our proposed approach can be defined as an abstract model with 5-tuple <B, *bel*, P, M, R>, where

1. $\boldsymbol{B} = \{b_1, b_2, \ldots b_n\}$ is a set of online auction bidders to be certified;
2. ***bel*:** $B \rightarrow [0, 1]$ is a scoring function**.** There is a degree of belief for every online auction bidder, representing the system's belief that a bidder is a shill or not.
3. $\boldsymbol{P} = \{p_1, p_2, \ldots, p_k\}$ is a set of bidders' properties, which can be considered as evidence either for shilling behaviors or normal bidding behaviors.
4. $\boldsymbol{M} = \{m: P \rightarrow [0,1]\}$ is a set of mass assignment functions which quantify every piece of evidence into a mass that supports either *shill* or *~shill.*
5. $\boldsymbol{R} = \{\theta, \varphi\}$ is the set of thresholds for making decisions on a bidder's certifications, where $\theta < \varphi$. The first element θ is the belief value threshold for determining if a bidder is a trusted bidder. If the value of $bel(shill_i)$ is below θ, the $bidder_i$ will be certified as a *Trusted Bidder*. The second element φ is the belief value threshold for determining shills, and it is larger than 0.5. If the value of $bel(shill_i)$ exceeds φ, the $bidder_i$ will be certified as a *Shill*. For any bidder, if the shilling score is between θ and φ, and *bel*(*shill*) is greater than or equal to *bel*(~*shill*), the certification of the bidder would be updated to *Suspect*.

Certifying a group of bidders B is to assign every bidder b_i in B a role to indicate the bidder's trustworthiness, i.e., deciding if a bidder is a *shill*, a *suspect*, or a *trusted bidder*. For any bidder's property $p_i \in P$, it can be utilized to either support a bidder is a legitimate bidder or a shill, depending on the nature of the property and the quantified value of the evidence. For example, if a bidder placed quite a few abnormal concurrent bids in a sellers' auction, it becomes evidence to support that the bidder is a shill. However, if a bidder places very few abnormal concurrent bids in online auctions, it should be considered as evidence to support the bidder is not a shill. Each property can only support a bidder for one state but not both. At the auction level, the decision boundary can be the average level of all auctions in the same category. If an auction's property value is significant (i.e., not close to the average value), the corresponding evidence can be considered as one to support that the auction involves shilling behavior or the auction is normal, depending on the value and the nature of the property. For example, suppose auctions in a certain category attracted 7.67 bids on average in the past 30 days. Now if an auction ends with 60 bids, we may consider that the auction involves shills. Since each property p_i for a bidder can only support one state, the rest of the belief from property p_i cannot commit to another state other than the universal set, i.e., the *frame of discernment.* Intuitively, the universal set, e.g., {*shill*, ~*shill*}, can be interpreted as *uncertainty* about any state. The ability to represent and quantify uncertainties is a key advantage of Dempster-Shafer theory. The BMA for the evidence that corresponds to property p_i in supporting shilling behavior can be represented as in Eq. (7), Eq. (8), and Eq. (9).

$$m_{pi}(shill) = \alpha * f \tag{7}$$

$$m_{pi}(\sim shill) = 0 \tag{8}$$

$$m_{pi}(U) = 1 - \alpha * f \tag{9}$$

where $0 < \alpha < 1$, and it is an adjusted value that can be understood as the strength of property p_i on determining if a bidder is a shill. The function f quantifies evidence for shill certification, where $0 < f \leq 1$.

The BMA for the evidence that corresponds to property p_j in supporting normal bidding behavior can be represented using Eq. (10), Eq. (11), and Eq. (12).

$$m_{pj}(shill) = 0 \tag{10}$$

$$m_{pj}(\sim shill) = \beta * g \tag{11}$$

$$m_{pj}(U) = 1 - \beta * g \tag{12}$$

where $0<\beta<1$, and it is an adjusted value that is the strength of property p_j on determining if a bidder is not a shill. The function g quantifies evidence for supporting the bidder is honest, where $0<g \leq 1$.

3.2. *The shill certification framework*

An automated shill certification system can play a significant role in maintaining trust among online auction users. The major task of our proposed shill certification system is to identify shills and recognize honest bidders. Figure 1 depicts the shill certification framework based on D-S theory.

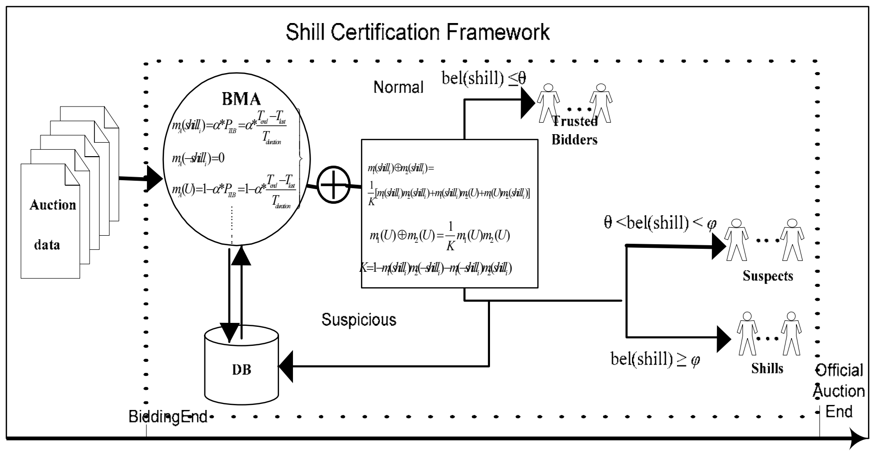

Fig. 1. Shill certification framework

Auction bidders are certified mathematically using a data fusion method that combines information from different aspects of bidders' behaviors and auction-level features. The certification process classifies all bidders into categories reflecting the likelihood of an "actual" shill. Initially, every bidder in the auction house is categorized as a *Trusted Bidder*. When the bidding process of an auction ends, the auction enters a shill certification stage, and the auction does not officially close until the certification procedure is complete. After an auction is officially closed with valid certifications for all bidders, the seller and the winner of the auction can proceed for further activities such as payment, shipping, and mutual feedback.

In the certification process, the monitored auction data along with historical statistical data stored in a database is the input to the basic mass assignment (BMA) module (as shown in Figure 1). Each bidder's behavior is checked and quantified based on the formulas that will be defined in Section 5.1. If the BMA module does not have sufficient belief to support a bidder is a shill, the bidder is classified as a *Trusted Bidder* immediately. Note that this piece of information is stored into the historical database for future use. On the other hand, if a bidder's bidding behavior shows any shill bidding properties, the pieces of evidence obtained from BMA module are combined together using Dempster's rule [14] (denoted as $\oplus$ in Figure 1). The results of the evidence combination process are belief values that indicate the likelihood of being a shill or a normal bidder. Needed information for computing the belief values, such as the other evidence for the same bidder in the same auction, can be fetched from the database. Once the belief values are calculated, the certification system updates each bidder's certification according to the certification assignment rules as shown in Figure 2.

bel(shill) $\geq \varphi$ => Shill
$\varphi >$ bel(shill) $> \theta$ => Suspect
bel(shill) $\leq \theta$ => Trusted Bidder

Fig. 2. Certification assignment rules

The threshold (φ) of certifying bidders as *Shill* should be fairly high to reduce the number of false positives. For the bidders that are certified as *Suspect*, the values of their *bel(shill)* must be lower than φ but still greater than the values of *bel*(~*shill*). This means that the evidence is not sufficient enough to support a bidder for being a shill, even though the bidder behaved more like a shill than an honest bidder. As a result, the bidder is assigned a certification of *Suspect*. When any additional independent evidence is available, the certification of *Suspect* shall be validated again. If a bidder's certification changes, the new certification is committed to the database. If a bidder's certification is labeled as *Shill*, the bidder is subject to further investigation and possible punishment, but the shill-handling step is outside the scope of this paper.

The required statistical data for computing the basic mass assignment includes information such as the bidder's total number of bids in a certain period of time, and the number of bids in the particular auction. Such information is stored in a historical database, and can be fetched when needed. The database is updated periodically when each certification process completes.

4. Shill Related Properties: Bid-level and Auction-level

There are two types of properties that can be used to provide evidence of shilling behaviors – those properties associated with a particular bidder, such as the time when he placed his last bid, and those properties associated with the auction itself, such as the total number of bids in the auction. The auction-level properties can be used as evidence to support that an auction involves shills; while the bid-level properties can be used as evidence to support that a bidder is a shill. Note that if an auction is suspected of involving shills by the evidence at the auction level, every bidder in the auction is considered as a shill suspect initially. In other words, if an auction-level property is used as evidence for shill, it is used as evidence supporting shill for every bidder in the auction. On the other hand, if the property is used as evidence for not shilling, all bidders in the auction get one more piece of evidence to support that they are honest. Therefore, when combining the evidence at the auction level with the evidence at the bid level, the auction-level evidence is just used as a piece of bid-level evidence.

In order to demonstrate the feasibility of our approach, we define several bidding properties and auction features that are used in our case study. Note that the list of possible properties we provide is not necessarily complete; providing such a complete list is beyond the scope of this paper.

4.1. *Bid-level properties*

Property TLB (Time of Last Bid). The time a bidder places his last bid in an auction can reflect the genuineness of the bidding purpose. Generally speaking, shill bidders typically avoid placing bids in a later stage of an auction in order to reduce the risk of winning. In other words, a bidder who places a bid in the late stage of an auction is more likely an honest bidder who aims to win the auction. This evidence can support either shill or normal bidder, depending on the relative time at which a bidder places his last bid. We quantify the relative time of such bids by Eq. (13).

$$TLB = \frac{T_{end} - T_{last}}{T_{duration}} \tag{13}$$

where T_{end} is the end time of the auction; T_{last} is the time when the monitored $bidder_i$ places his last bid; and $T_{duration}$ is the duration of the auction. Thus, in terms of this particular evidence, the likelihood of a bidder being a shill increases as P_{TLB} increases. The earlier such a (last) bid is placed in an auction, the more suspicious is the bidder. When the last bid is placed in the final stage of the auction (we define it as $[0.9T_{end}, T_{end}]$, following the definition in [15]), this information can be considered as evidence to support that the bidder is honest.

Property CBA (Concurrent Bid Activity). Shill bidders are not bargain hunters, while most legitimate bidders are. Because shill bidders' purpose is different from that of legitimate bidders, shill bidders typically do not favor items with lower prices. They may place bids in an auction that has a higher current bidding price rather than in some concurrent auctions that have *lower* current bidding prices [6]. We consider bidders placing such abnormal bids as candidates of shills. We capture this indicator of shilling behavior as the percentage of abnormal concurrent bids (*ACB*) placed by a bidder with respect to a particular seller as given by Eq. (14).

$$CBA = \frac{ACB_{(i,j)}}{\sum\limits_{j=0, j \neq i}^{n} ACB_{(i,j)}} \tag{14}$$

where $ACB_{(i,j)}$ is the number of abnormal concurrent bids that $bidder_i$ has placed in auctions hosted by $seller_j$.

Property AF (Average Feedback): A feedback score is an indicator of an online auction user's reputation, which can be used as a predictor for the user's future behavior. Generally speaking, a positive rating increases a user's feedback score, and a negative rating decreases the feedback score. Since a high feedback score is important for a bidder to gain trust from sellers and other bidders, auction users try to maintain good reputations, which may possibly have been accumulated over a long period of time. Therefore, users with good feedback histories normally would not take the risk of being a shill. On the other hand, because shills seldom win auctions, they do not accumulate much feedback. This source of evidence can be quantified by comparing a bidder's feedback score against the average feedback (*AF*) score of all users in the same category. When a bidder's feedback score is less than the average, the evidence to support that a bidder is a shill is calculated by Eq. (15.1). For the same reason, if a bidder's feedback score is greater than or equal to the average, property *AF* shall be considered as evidence for normal bidder. In this case, the mass for it should be calculated according to Eq. (15.2).

$$AF = 1 - \frac{FB_i}{FB_{avg}} \qquad (15.1) \qquad\qquad AF = 1 - \frac{FB_{avg}}{FB_i} \qquad (15.2)$$

where FB_{avg} is the average feedback score of all bidders in the same category of auctions and FB_i is the feedback score of $bidder_i$.

Property BIA (Bidding Increment Activity): The minimum increment of an auction is usually set up by an auction house before the auction begins. Within different price ranges, the auction house requires different minimum increments. Typically, the minimum increment increases as the price range level grows. If a bidder wants to outbid another buyer, the bidder must place a bid that at least equals the current price plus a minimum increment. Generally speaking, a bargain hunter usually tries to place bids with the minimum increment so as to win the item at a price as low as possible, while a shill often adds a large increment in an attempt to raise the price quickly. As we have observed, shills tend to place such a larger increment at an early stage of an auction, and before the price reaches the normal price range, where the early stage of an auction is defined as the first quarter of the auction time, following the definition in [15]. Typically there are only a few bids placed, but a shill bidder may be eager to drive up the price as early as possible. This is because if a shill places quite a large bid at the final stage of an auction, the price would likely drive potential buyers away and may cause the shill to become the auction winner. This is the worst situation, which shills try to avoid. When quantifying the evidence, we compare the average increment a bidder placed with the minimum increment, and only the increments placed before the final stage of an auction are considered.

$$BIA = \frac{\sum_{l=0}^{s} \frac{m * MIN_l}{\sum_{i=1}^{m} I_i}}{s} \qquad (16)$$

where I_i is a bidder's i^{th} increment, s is the total number of different minimum increments in s different price ranges, m is the total number of increments the bidder added in a price range, and MIN_l represents the minimum increment of the l^{th} price range defined in the auction rules. For example, an auction house sets the minimum increment rule as follows. When the current price is between \$0.01 and \$0.99, the required minimum increment is \$0.05; the minimum increment is \$0.25, when the current price is between \$1.00 and \$4.99; when the current price is between \$5.00 and \$24.99, the required minimum increment is \$0.50, and so on. Suppose the final price of an auction is \$24.99 and the average increments that a bidder placed in the three different prices ranges are \$0.50, \$0.9, and \$2.00, respectively. Then the value of *BIA* for the bidder is calculated as the following:

$$BIA = \frac{\frac{0.05}{0.5} + \frac{0.25}{0.9} + \frac{0.5}{2.00}}{3} = 0.21$$

This result shows that the bidder might be a shill. The likelihood of a bidder being a normal bidder increases as the value of *BIA* decreases. But, when a bidder's increment exceeds a pre-specified value, the *BIA* property would be used as evidence to support that the bidder is a shill.

Property WPB (Wins Per Bid). Normal bidders usually win several auctions during a period of time, while shill bidders do not win. This property can be measured by Wins Per Bid (*WPB*). We compare the bidder's *WPB* value for a specific bidder, $WPB_{(i,j)}$ as shown in Eq. (17.1), with the bidder's overall *WPB* value as given by Eq. (17.2).

$$WPB_{(i,j)} = \frac{NOW_{(i,j)}}{NOB_{(i,j)}} \quad (17.1) \qquad WPB = \frac{\sum_{j=1, j\neq i}^{j=n} NOW_{(i,j)}}{\sum_{j=1, j\neq i}^{j=n} NOB_{(i,j)}} \quad (17.2)$$

where $NOW_{(i,j)}$ is $bidder_i$'s number of wins in the auctions that were hosted by $seller_j$, and $NOB_{(i,j)}$ is the number of bids that $bidder_i$ has placed in $seller_j$'s auctions. If the bidder's $WPB_{(i,j)}$ is lower than *WPB*, this information will be used as evidence to support that the bidder is a shill. Otherwise, this is a source of evidence that supports the bidder is not a shill.

Property AS (Affinity for Sellers). Shills usually have a close affinity for a particular seller. A normal bidder may place bids in different sellers' auctions, while a shill tends to participate in a great number auctions conducted by a particular seller who may have collaboration with the shill. The degree of abnormality of a bidder's bid activity is quantified by the percent of participation for a seller's auctions, given by Eq. (18). The abnormality increases as the bidding frequency grows.

$$AS_{(i,j)} = \frac{AUC_{(i,j)}}{NOA_j} \quad (18)$$

where $AUC_{(i,j)}$ is the number of auctions hosted by $seller_j$ and participated in by $bidder_i$, and $NOA_{(i,j)}$ is the total number of auctions hosted by $seller_j$ in a certain period of time. If a bidder's $AS_{(i,j)}$ score is low, the *AS* property will be used as evidence to support that the bidder is a normal bidder. Otherwise, this is a source of evidence to support that the bidder is a shill.

4.2. *Auction-level properties*

The properties specified previously are often considered as bid-level properties. Next we specify shill-relevant properties from the auction level. These properties can either support auctions with shill bidding or support valid auctions.

Property NB (Number of Bids): Auctions with shills usually end with more bids on average than those without shills. When comparing auctions of similar duration and items for bid, shills tend to outbid the legitimate bids frequently until the price reaches their expected value, or when the risk of winning the auction becomes high. The bids that the shills stimulated and placed contribute to the extra amount of bids in the auction. If the number of bids in an auction is more than the average number of bids in the same category, there is a chance that the auction involves shills. In this case, Property *NB* can be quantified as in Eq. (19.1). Similarly, if the number of bids in an auction is smaller than the average, the possibility that the auction employed shill bids is reduced. In fact, the fewer bids an auction has, the more possibility the seller is honest and did not employ shills. In this case, the degree of the auction being normal is given by Eq. (19.2).

$$NB = 1 - \frac{NB_{avg}}{NB_k} \qquad (19.1) \qquad\qquad NB = 1 - \frac{NB_k}{NB_{avg}} \qquad (19.2)$$

where NB_k is the number of bids that are placed in the monitored auction and NB_{avg} is the average number of bids that were placed in an auction of the same product over the last 30 days.

Property SP (Starting Price): The starting price (*SP*) of an auction that involves shills is usually less than the average starting price of auctioned items in the same category. In other words, the higher the starting bid (compared to book value), the less possibility that the auction involves shills. Conversely, if the starting bid is much less than the book value, it is more likely that the auction involves a shill. This indicator is explained and tested in [16]. In auction houses, the commission fee is partly based on the starting price. So by lowering the starting price of an auction, sellers can save money on the commission fee. If an auction's starting price is higher than average, this might indicate that the seller has no intention to engage in shilling. Because, otherwise, if the seller planned to use shill bidding, he would not have to start the auction at a higher price, which incurs a higher listing fee, i.e., a part of the commission fee. Therefore, the higher the starting price, the more possibility the auction is normal, as given in Eq. (20.1). In contrast, if an auction's starting price is lower than average, it is possible that the auction involves shills. The property to support a shill auction is given by Eq. (20.2)

$$SP = \frac{SP_{avg}}{SP_k} \qquad (20.1) \qquad\qquad SP = \frac{SP_K}{SP_{avg}} \qquad (20.2)$$

where SP_k is the starting price of the monitored auction, and SP_{avg} is the average starting price for the same product over the last 30 days.

Since any of the properties mentioned above involve uncertainties, we now propose to employ D-S theory to reduce the uncertainties and conflicts in incriminating shills.

5. Shill Certification

5.1. *Basic mass assignment*

As we mentioned earlier, the shill certification process employs a mathematical theory, D-S theory of belief functions, to represent the uncertainties of evidence pertaining to different hypotheses. Consider the states {*shill*}, {~*shill*}, and Θ. We now provide basic mass assignments (BMA) for evidence *TLB, CBA, WPB, BIA, AF, AS, NB, and SP,* which were described in Section 4.

BMA for Evidence TLB: The basic mass as in (21.1) is assigned to ~*shill* only if a bidder places his last bid in the final stage of an auction. In this stage, a shill only places bids occasionally and very carefully in order to avoid winning the auction. When a bidder is detected to have a tendency to stop bidding earlier, the bidder should be identified as a shill candidate and the mass is assigned to *shill*, as given by Eq. (21.2).

$$\left.\begin{aligned} &m_{TLB}(shill) = 0 \\ &m_{TLB}(\sim shill) = \beta_{TLB} * (1 - TLB) = \beta_{TLB} * \left(1 - \frac{T_{end} - T_{last}}{T_{duration}}\right) \\ &m_{TLB}(\Theta) = 1 - \beta_{TLB} * \left(1 - \frac{T_{end} - T_{last}}{T_{duration}}\right) \end{aligned}\right\} \qquad (21.1)$$

$$\left.\begin{aligned} &m_{TLB}(shill) = \alpha_{TLB} * TLB = \alpha_{TLB} * \frac{T_{end} - T_{last}}{T_{duration}} \\ &m_{TLB}(\sim shill) = 0 \\ &m_{TLB}(\Theta) = 1 - \alpha_{TLB} * \frac{T_{end} - T_{last}}{T_{duration}} \end{aligned}\right\} \qquad (21.2)$$

BMA for Evidence CBA: When a bidder passes up many chances to place a lower bid for an item in a concurrent auction, the bidder is considered as a shill candidate. On the other hand, if a bidder places very few abnormal concurrent bids, *CBA* is used as evidence to support the state of ~*shill*. Eq. (22.1) and Eq. (22.2) provide the basic mass assignments for *shill* and ~*shill*, respectively.

$$\left.\begin{aligned} m_{CBA}(shill) &= \alpha_{CBA} * CBA = \alpha_{CBA} * \frac{ACB_{(i,j)}}{\sum_{i=0, i \neq j}^{n} ACB_{(i,j)}} \\ m_{CBA}(\sim shill) &= 0 \\ m_{CBA}(\Theta) &= 1 - \alpha_{CBA} * CBA = 1 - \alpha_{CBA} * \frac{ACB_{(i,j)}}{\sum_{i=0, i \neq j}^{n} ACB_{(i,j)}} \end{aligned}\right\} \quad (22.1) \qquad \left.\begin{aligned} m_{CBA}(shill) &= 0 \\ m_{CBA}(\sim shill) &= \beta_{CBA} \\ m_{CBA}(\Theta) &= 1 - \beta_{CBA} \end{aligned}\right\} \quad (22.2)$$

BMA for Evidence WPB: When a bidder's winning ratio (i.e., Wins Per Bid) in a specific bidder's auction is lower than his normal winning ratio, the bidder is likely to be a shill bidder. The basic mass assignment for the state that the bidder is a shill is given by Eq. (23.1). If the bidder's winning ratio for a specific bidder is greater than or equal to the overall average value, the basic mass for ~*shill* can be calculated by Eq. (23.2).

$$\left.\begin{aligned} m_{WPB}(shill) &= \alpha_{WPB} * (1 - WPB_{(i,j)}) = \alpha_{WPB} * \left(1 - \frac{NOW_{(i,j)}}{NOB_{(i,j)}}\right) \\ m_{WPB}(\sim shill) &= 0 \\ m_{WPB}(\Theta) &= 1 - \alpha_{WPB} * \left(1 - \frac{NOW_{(i,j)}}{NOB_{(i,j)}}\right) \end{aligned}\right\} \quad (23.1)$$

$$\left.\begin{aligned} m_{WPB}(shill) &= 0 \\ m_{WPB}(\sim shill) &= \beta_{WPB} * WPB_{(i,j)} = \beta_{WPB} * \frac{NOW_{(i,j)}}{NOB_{(i,j)}} \\ m_{WPB}(\Theta) &= 1 - \beta_{WPB} * \frac{NOW_{(i,j)}}{NOB_{(i,j)}} \end{aligned}\right\} \quad (23.2)$$

BMA for Evidence BIA: When a bidder is detected to place bids with increments that are much greater than the minimum increment before the final stage of an auction, the bidder should be identified as a shill candidate. The BMA is given by Eq. (24.1). On the other hand, if a bidder places bids with only small increments, the evidence should support that the bidder is an honest bidder. Eq. (24.2) shows how to calculate the basic mass assignment for bidder *i* being honest.

$$\left.\begin{array}{l} m_{BIA}(shill) = \alpha_{BIA} * (1 - BIA) = \alpha_{BIA} * (1 - \dfrac{\sum\limits_{l=0,t=0}^{s,0.9T} \dfrac{m * MIN_l}{\sum\limits_{i=1}^{m} I_i}}{s}) \\ m_{BIA}(\sim shill) = 0 \\ m_{BIA}(\Theta) = 1 - \alpha_{BIA} * (1 - \dfrac{\sum\limits_{l=0,t=0}^{s,0.9T} \dfrac{m * MIN_l}{\sum\limits_{i=1}^{m} I_i}}{s}) \end{array}\right\} \quad (24.1)$$

$$\left.\begin{array}{l} m_{BIA}(shill) = 0 \\ m_{BIA}(\sim shill) = \beta_{BIA} * BIA = \beta_{BIA} * \dfrac{\sum\limits_{l=0,t=0}^{s,0.9T} \dfrac{m * MIN_l}{\sum\limits_{i=1}^{m} I_i}}{s} \\ m_{BIA}(\Theta) = 1 - \beta_{BIA} * \dfrac{\sum\limits_{l=0,t=0}^{s,0.9T} \dfrac{m * MIN_l}{\sum\limits_{i=1}^{m} I_i}}{s} \end{array}\right\} \quad (24.2)$$

BMA for Evidence AF: Users with good feedback histories are usually less likely shills. If a bidder's feedback score is greater than or equal to the average of all users' in the same category, evidence *AF* supports that the bidder is a normal bidder. In this case, the basic mass is assigned as in Eq. (25.1). Otherwise, if a bidder's feedback score is lower than the average feedback score of all users, *AF* should be counted as evidence to support shilling behavior. In this case, the basic mass is assigned using Eq. (25.2).

$$\left.\begin{array}{l} m_{AF}(shill) = 0 \\ m_{AF}(\sim shill) = \beta_{AF} * AF = \beta_{AF} * (1 - \dfrac{FB_{avg}}{FB_i}) \\ m_{AF}(\Theta) = 1 - \beta_{AF} * (1 - \dfrac{FB_{avg}}{FB_i}) \end{array}\right\} \quad (25.1)$$

$$\left.\begin{array}{l} m_{AF}(shill) = \alpha_{AF} * AF = \alpha_{AF} * (1 - \dfrac{FB_i}{FB_{avg}}) \\ m_{AF}(\sim shill) = 0 \\ m_{AF}(\Theta) = 1 - \alpha_{AF} * AF = 1 - \alpha_{AF} * (1 - \dfrac{FB_i}{FB_{avg}}) \end{array}\right\} \quad (25.2)$$

BMA for Evidence AS: A bidder may place bids in auctions that were hosted by the same seller; however, if the bidder placed bids in most of that seller's auction, the bidder shall be suspected as a shill working with the seller. When $AS_{(i,j)}$ is within the normal range, property *AS* suggests the bidder is normal and AS is used as non-shill evidence, as in Eq. (26.1). If $AS_{(i,j)}$ is greater than a threshold, say 50%, the property *AS* will be used as evidence to support that the bidder is a shill, as in Eq. (26.2)

$$\left.\begin{aligned} m_{PS}(shill) &= 0 \\ m_{PS}(\sim shill) &= \alpha_{PS} * (1-PS) = \alpha_{PS} * (1-\frac{AUC_{(i,j)}}{NOA_j}) \\ m_{PS}(\Theta) &= 1-\alpha_{PS} * (1-\frac{AUC_{(i,j)}}{NOA_j}) \end{aligned}\right\} (26.1) \qquad \left.\begin{aligned} m_{PS}(shill) &= \beta_{PS} * PS = \beta_{PS} * \frac{AUC_{(i,j)}}{NOA_j} \\ m_{PS}(\sim shill) &= 0 \\ m_{PS}(\Theta) &= 1-\beta_{PS} * \frac{AUC_{(i,j)}}{NOA_j} \end{aligned}\right\} (26.2)$$

BMA for Evidence NB: When the number of bids in an auction is greater than the average number of bids in auctions in the same category, that auction might be suspected of having shill bids. For the same reason, if an auction completes with number of bids less than the average, the auction might not be compromised by shills. When NB_{avg} is smaller than NB_k, property *NB* suggests this auction may involve shills. The possibility of shills can be calculated using Eq. (27.1). When NB_k is smaller than NB_{avg}, evidence *NB* suggests the auction is normal. Then we assign the value to *NB*, the non-shill evidence, as in Eq. (27.2).

$$\left.\begin{aligned} m_{NB}(shill) &= \alpha_{NB} * NB = \alpha_{NB} * (1-\frac{NB_{avg}}{NB_k}) \\ m_{NB}(\sim shill) &= 0 \\ m_{NB}(\Theta) &= 1-\alpha_{NB} * (1-\frac{NB_{avg}}{NB_k}) \end{aligned}\right\} (27.1) \qquad \left.\begin{aligned} m_{NB}(shill) &= 0 \\ m_{NB}(\sim shill) &= \beta_{NB} * NB = \beta_{NB} * (1-\frac{NB_k}{NB_{avg}}) \\ m_{NB}(\Theta) &= 1-\beta_{NB} * NB = 1-\beta_{NB} * (1-\frac{NB_k}{NB_{avg}}) \end{aligned}\right\} (27.2)$$

BMA for Evidence SP: If the starting price of an auction is lower than normal, it may indicate that a shill might try to drive up the price in the early stage, which results in a hidden higher starting price. For the same reason, if the starting price of an auction is higher than normal, the auction may have less chance of involving shill bids. Eq. (28.1) and Eq. (28.2) shows how to quantify evidence *SP*. When SP_k is less than SP_{avg}, property *SP* suggest we trust the auction. We assign *SP* a mass, as given by Eq. (28.1). Otherwise, we assign a mass to shill evidence *SP*, as given by Eq. (28.2).

$$\left.\begin{aligned} m_{SP}(shill) &= \alpha_{SP} * (1-SP) = \alpha_{SP} * (1-\frac{SP_k}{SP_{avg}}) \\ m_{SP}(\sim shill) &= 0 \\ m_{SP}(\Theta) &= 1-\alpha_{SP} * (1-\frac{SP_k}{SP_{avg}}) \end{aligned}\right\} (28.1) \qquad \left.\begin{aligned} m_{SP}(shill) &= 0 \\ m_{SP}(\sim shill) &= \beta_{SP} * (1-SP) = \beta_{SP} * (1-\frac{SP_{avg}}{SP_k}) \\ m_{SP}(\Theta) &= 1-\beta_{SP} * (1-\frac{SP_{avg}}{SP_k}) \end{aligned}\right\} (28.2)$$

5.2. Evidence combination

Once the basic probability assignments are obtained, different pieces of evidence are combined in a consistent manner to provide a more complete assessment on shill bidding, and thus it reduces the uncertainties involved in individual evidence. The evidence fusion procedure can be carried out using Dempster's combination rule. The corresponding rules of combining evidence for *shill* and *~shill* are listed as the following:

$$belief(shill_i) = m(shill_i) \tag{29}$$
$$m(shill_i) = m_1(shill_i) \oplus m_2(shill_i) \oplus \cdots \oplus m_n(shill_i) \tag{30}$$
$$belief(\sim shill_i) = m(\sim shill_i) \tag{31}$$
$$m(\sim shill_i) = m_1(\sim shill_i) \oplus m_2(\sim shill_i) \oplus \cdots \oplus m_n(\sim shill_i) \tag{32}$$

To combine multiple pieces of evidence for shill bidding behaviors, we can compute *bel(u_i)* by combining any pair of evidence first, and then combining the result with the remaining third one, forth one, and so on, For instance, Eq. (33) – Eq. (37) combines evidence 1 and evidence 2.

$$m_1(X) \oplus m_2(X) = \frac{\sum_{E_1 \cap E_2 = X \neq \varnothing} m(E_1) * m(E_2)}{1 - \sum_{E_1 \cap E_2 = \varnothing} m(E_1) * m(E_2)} \tag{33}$$

$$m_1(shill_i) \oplus m_2(shill_i) = \frac{m_1(shill_i) * m_2(shill_i) + m_1(shill_i) * m_2(U) + m_1(U) * m_2(shill_i)}{1 - k} \tag{34}$$

$$m_1(\sim shill_i) \oplus m_2(\sim shill_i) = \frac{m_1(\sim shill_i) * m_2(\sim shill_i) + m_1(\sim shill_i) * m_2(U) + m_1(U) * m_2(\sim shill_i)}{1 - k} \tag{35}$$

$$m_1(U) \oplus m_2(U) = \frac{m_1(U) * m_2(U)}{1 - k} \tag{36}$$

where $k = m_1(shill_i) * m_2(\sim shill_i) + m_1(\sim shill_i) * m_2(shill_i)$ (37)

The factor k is a measure of the amount of conflict between the two masses. The value of *bel(u_i)* indicates the degree of credibility of u_i.

6. Case Study and Result Analysis

The method proposed in this paper has been successfully examined using real online auction data from eBay. Before presenting the results and our analysis, a few issues related to implementing the experiments need to be clarified.

First, the parameters used in this approach need to be specified and fixed in order to provide a consistent result. The alpha value for each piece of evidence used in the paper is set subjectively based on different levels of importance as we observed. For example, the evidence of Concurrent Bid Activity and the evidence of Affinity for Seller are considered more important than other evidence. Therefore, we assign them a higher weight. In addition, the evidence of Time to Last Bid is not likely to be very reliable, by itself, to determine a shill in comparison to other evidence. Yet the later a bidder places a bid in an auction, the less suspicious it is that the bidder is a shill since late bid increases the risk of winning. Based on such above observations, we set the alpha value for each piece of evidence according to the importance of the evidence in determining a shill. We set α_{TLB}, α_{AS}, α_{CBA}, α_{WPB}, α_{AF}, α_{BIA}, α_{NB}, α_{SP} as 0.6, 0.95, 0.95, 0.9, 0.8, 0.7, 0.8, 0.8 and 0.8, respectively. We tested the alpha values on a set of training data collected from eBay, and observed that on average the certification results were as desired. In future work, we plan to design an approach, such as designing a neural network, to learn the alpha values automatically. Meanwhile, we set $\beta = \alpha$ for each piece of evidence so that the beta values can be trained easily and appropriately.

6.1. *Data collection*

The data used in our case study was collected from a recent auction on eBay with the title "Microsoft Xbox 360 Complete System & 20G Hard Drive." A detailed bidding history of the auction is shown in Figure 3. To protect the privacy of bidders, only symbolic IDs instead of the bidders' IDs are shown (along with the bidder's reputation score, shown in parenthesis). The detailed description of the item is provided below, in Table 1.

Table 1 Microsoft Xbox 360 Pro System - Game console - 20 GB

Description	
Model:	Microsoft Xbox 360
Hard Drive Capacity: 20GB	
Features	
Audio Output:	Surround Sound
Video Output:	ATI Xbox 360 - 256-bit - 2D/3D graphics acceleration
Max.	1920 x 1080
Connections	
1 x AV cable port, 3 x USB 2.0, 1 x Ethernet (RJ-45)	

XBOX 360 Auction Bidding History						
	Bidders:12	Bids:61		Time Ended:MAY-07-09 11:58:07 PDT		
e***e	US$167.50	May-07-09 11:57:54 PDT		6***o	US$60.00	May-06-09 19:11:53PDT
o***i	US$165.00	May-07-09 11:57:58 PDT		p***k	US$60.00	May-06-09 19:15:27PDT
e***e	US$160.00	May-07-09 11:57:27 PDT		o***i	US$58.00	May-06-09 19:11:32PDT
o***i	US$160.00	May-07-09 11:57:33 PDT		6***o	US$56.00	May-06-09 19:11:15PDT
o***i	US$155.00	May-07-09 11:57:05 PDT		a***l	US$55.00	May-06-09 19:10:36PDT
o***i	US$150.00	May-07-09 11:56:34 PDT		6***o	US$55.00	May-06-09 19:10:59PDT
o***i	US$146.00	May-07-09 11:56:10 PDT		o***i	US$52.00	May-06-09 19:08:53PDT
s***h	US$143.50	May-07-09 11:55:39 PDT		6***o	US$50.00	May-06-09 17:57:50PDT
o***i	US$142.00	May-07-09 11:54:47 PDT		o***i	US$50.00	May-06-09 19:08:38PDT
s***h	US$138.50	May-07-09 11:41:07PDT		o***i	US$45.00	May-06-09 19:08:53PDT
f***a	US$136.00	May-07-09 11:02:16 PDT		o***i	US$42.00	May-06-09 19:08:24PDT
s***h	US$133.50	May-07-09 10:47:42 PDT		o***i	US$40.00	May-06-09 19:08:11PDT
o***i	US$131.00	May-07-09 10:35:33 PDT		o***i	US$38.00	May-06-09 19:07:54PDT
s***h	US$128.50	May-07-09 09:50:43 PDT		p***p	US$35.00	May-06-09 19:07:42PDT
s***l	US$128.00	May-07-09 09:45:58 PDT		i***e	US$30.00	May-06-09 12:17:49PDT
s***h	US$123.50	May-07-09 09:50:02 PDT		p***p	US$30.00	May-06-09 13:28:46PDT
s***h	US$118.50	May-07-09 09:26:31 PDT		P***p	US$25.00	May-05-09 17:29:14PDT
s***l	US$116.00	May-07-09 06:45:20 PDT		n***0	US$20.00	May-05-09 18:11:46PDT
o***i	US$115.00	May-07-09 06:08:34 PDT		p***k	US$13.00	May-05-09 17:33:43PDT
o***i	US$110.00	May-07-09 06:08:05 PDT		p***k	US$12.00	May-05-09 17:32:07PDT
o***i	US$102.00	May-07-09 06:07:42 PDT		p***k	US$11.00	May-05-09 17:31:45PDT
s***l	US$100.00	May-07-09 20:06:30 PDT		p***k	US$10.00	May-05-09 13:39:28PDT
o***i	US$98.00	May-07-09 06:07:25 PDT		P***p	US$10.00	May-05-09 17:29:06PDT
o***i	US$95.00	May-07-09 06:07:09 PDT		p***p	US$5.00	May-04-09 19:49:57PDT
o***i	US$90.00	May-07-09 06:06:53 PDT		p***k	US$5.00	May-05-09 13:38:05PDT
o***i	US$88.00	May-07-09 06:06:36 PDT		p***k	US$3.00	May-05-09 13:37:51PDT
6***o	US$86.00	May-07-09 05:28:00 PDT		p***k	US$2.50	May-05-09 13:37:34PDT
p***p	US$75.00	May-06-09 21:36:33 PDT		v***i	US$1.00	May-04-09 18:33:24PDT
p***p	US$70.00	May-06-09 21:36:25 PDT		p***p	US$1.00	May-04-09 19:49:47PDT
p***p	US$67.00	May-06-09 21:36:12 PDT		p***p	US$0.50	May-04-09 19:49:36PDT
p***p	US$65.00	May-06-09 21:15:14PDT		Starting Price	US$0.01	May-04-09 11:58:07PDT

Fig.3. Bidding history

6.2. *Data processing*

The record for our case study is shown in Figure 4. Information about the auction, such as the detailed bidding history, can be obtained directly from the auction web pages. Most of the statistical data can be obtained from posted auction information and eBay web services. For instance, via eBay web services, we collected the 30-day average final price of all Xbox auctions that has the same options and it is $138.94. We also obtained that the average starting price of this kind of auction is $40.64; the average number of bids per auction is 7.67; and the average feedback score for bidders bidding actively in this category is 101.98. The statistics is updated periodically, but for the same category of items, it is constant in a short period of time, e.g., 1 day. Besides, some of the detailed information about the seller and bidders, such as feedback score and links to finished auctions, can be captured from their posted profiles.

We investigated auctions hosted by a particular seller during the past month, and the bidding history of every bidder who participated in this auction. The historical information is statistically processed. We counted each bidder's number of wins, total number of bids, total number of auctions participated, bid activity with the same seller, feedback score, number of abnormal concurrent bids, and etc. The statistical results are shown in Table 2. The basic masses assigned for evidence specified in Section 5.1 are shown in Table 3 and Table 4.

Table 2. Statistical data

Bidder	Bids in this aucti-on	Bid Activity with the seller[1]	Items bid on	Times bid on this seller's auction[2]	Same category sellers (all sellers) placed bid[3]	Total bids	Wins	Feed-back score	Num. of ACB[4]	Time from last bid to the end of auction[5]	Increment_{AVG}			
											$1	$0.5	$0.25	$0.05
e***e	2	1%	88	1	1(30)	112	2	642	0	13	0	0	0	0
o***i	21	79%	10	6	2(3)	82	1	2	11	9	6	0	0	0
s***h	6	100%	1	1	1(1)	6	0	0	0	148	0	0	0	0
f***a	1	100%	1	1	1(1)	1	0	0	0	3351	0	0	0	0
s***l	3	30%	120	30	4(6)	283	5	27	71	7929	$40	0	0	0
6***o	5	100%	3	3	1(1)	11	1	3	0	23407	$6.2	0	0	0
p***p	11	31%	5	1	5(5)	35	1	8	0	51094	$8.33	$10	$4	$0.495
p***k	8	83%	5	2	2(2)	30	0	0	1	60610	0	$11	0	0
a***l	1	7%	7	1	2(6)	17	0	20	0	60451	$3	0	0	0
i***e	1	100%	1	1	1(1)	1	0	7	0	85218	$5	0	0	0
n***0	1	50%	7	3	2(5)	24	0	8	0	150381	0	$19.5	0	0
v***i	1	53%	3	1	1(3)	3	0	0	0	235483	0	0	0	$0.99

1. This shows the percentage of all bids from this bidder that went to this specific seller. 2. In the last 30 days, the number of auctions that were hosted by the seller the bidder participated in. The seller's total number of Xbox Game System auctions is 36. 3. This shows that the bidder placed bids for how many different games system sellers. The number in the parenthesis is the total number of sellers the bidder has placed bid for, no matter what categories of goods they sell. 4. ACB stands for abnormal concurrent bid; 5. time from last bid to the end of auction (in seconds) = The duration of the auction - duration of a bidder's last bid since the auction begins. eBay only provides a bidder's bid history for the last 30 days.

Table 3. The basic mass assignments for bid-level evidence TLB, AS, and ACB

Bidder	m_{TLB}(shill)	m_{TLB}(~shill)	m_{TLB}(U)	m_{AS}(shill)	m_{AS}(~shill)	m_{AS}(U)	m_{CBA}(shill)	m_{CBA}(~shill)	m_{CBA}(U)
e***e	0	0.5	0.5	0	0.9236	0.0764	0	0.95	0.05
O***i	0	0.9	0.1	0	0.7917	0.2083	0.3458	0	0.6542
s***h	0.0003	0.6	0.4	0	0.9236	0.0764	0	0.95	0.05
f***a	0	0.592	0.408	0	0.9236	0.0764	0	0.95	0.05
s***l	0.0184	0	0.9816	0.7917	0	0.2083	0.8693	0	0.1308
6***o	0.0542	0	0.9458	0	0.8708	0.1292	0	0.95	0.05
P***P	0.1183	0	0.8817	0	0.9236	0.0764	0	0.95	0.05
P***k	0.1403	0	0.8597	0	0.8972	0.1028	0	0.95	0.05
a***l	0.1399	0	0.8601	0	0.9236	0.0764	0	0.95	0.05
i***e	0.1973	0	0.8027	0	0.9236	0.0764	0	0.95	0.05
n***0	0.3481	0	0.6519	0	0.8708	0.1292	0	0.95	0.05
v***i	0.5451	0	0.4549	0	0.9236	0.0764	0	0.95	0.05

RECORD							
Microsoft Xbox 360 System-Game Console-20GB Hard Drive							
AUCTION INFO	Starting price	Num. of Bids	Num. of Bidders	Duration	Start time	End time	Winning Bid
	0.01	42	12	3day(259200 s)	lay-04-09 11:58:07 PI	May-07-09 11:58:07 PD	US $167.50
HISOTRICAL STATISTICS	Average price	Average starting price	Average number of Bids	Average Feedbacks	Increment$_{MIN}$		
	138.94	40.64	7.67	101.98	($0.01-$0.99)$0.05; ($1.00-$4.99)$0.25;($5.00-$24.99)$0.5;($25.00-$99.99)$1.00;($100-$249.99)$2.5;($250-$499.99)$5.00;		
SELLER INFO	Seller ID	Num. of auctions	Bids attracted in video Game	Bids/Auction	Num. of categories	Feedback(percentage)	Active Since
	bascoo1998	36	909	25.25	7	9(90.9%)	MAY-3-2009

Fig. 4. History record

Table 4. The basic mass assignments for bid-level evidence WPB, BIA, and AF

Bidder	m_{WPB}(shill)	m_{WPB}(~shill)	m_{WPB}(U)	m_{BIA}(shill)	m_{BIA}(~shill)	m_{BIA}(U)	m_{AF}(shill)	m_{AF}(~shill)	m_{AF}(U)
e***e	0	0.45	0.55	0	0.8	0.2	0	0.58884	0.4112
O***i	0	0.0139	0.9861	0	0.0333	0.9667	0.6863	0	0.3137
s***h	0	0	1	0	0.8	0.2	0.7	0	0.3
f***a	0	0	1	0	0.8	0.2	0.7	0	0.3
s***l	0	0.0482	0.9518	0.795	0	0.205	0.51467	0	0.4853
6***o	0.9	0	0.1	0	0.0323	0.9677	0.6794	0	0.3206
P***P	0	0.0829	0.9171	0	0.0667	0.9333	0.6451	0	0.3549
P***k	0	0	1	0	0.0091	0.9909	0.7	0	0.3
a***l	0	0	1	0	0.066	0.934	0.56272	0	0.4373
i***e	0	0	1	0	0.04	0.96	0.65195	0	0.348
n***0	0	0	1	0.7949	0	0.2051	0.6451	0	0.3549
v***i	0	0	1	0	0.0101	0.9899	0.7	0	0.3

Besides evidence from bid level, we also use evidence from the auction level, such as starting price (SP) and number of bids (NB), to facilitate the certification of auction bidders. According to Eq. (27.1), Eq. (27.2), Eq, (28.1), and Eq. (28.2), the masses for SP and NB are assigned as shown in Table 5.

Table 5. Basic mass assignments for auction-level evidence

m_{NB}(shill)	m_{NB}(~shill)	m_{NB}(U)	m_{SP}(shill)	m_{SP}(~shill)	m_{SP}(U)
0.65360	0	0.3464	0.799	0	0.201

The shill certification results are shown in Table 6. Recall that functions *bel* and *pl* define belief and plausibility, respectively, as presented in Section 2. Each bidder is recognized with one of the three certifications: *Shill*, *Suspect*, and *Trusted Bidder*. The certification levels ensure that each bidder is certified and fraudulent bidders are identified. In this example, we set R= {0.95, 0.5}. To reduce the number of false positives generated from our proposed approach, the shill threshold φ should be sufficiently high. These certification results are assigned in accordance with the rule shown in Figure 5.

Table 6. Shill certification results

Bidder	bel(shill)	pl(shill)	bel(~shill)	pl(~shill)	Results
e***e(642)	0.00115	0.00124	0.99876	0.99885	Trusted Bidder
O***i(2)	0.57803	0.58641	0.41359	0.42197	Suspect
s***h(0)	0.01398	0.01428	0.98572	0.98602	Trusted Bidder
f***a(0)	0.01440	0.01471	0.98529	0.98560	Trusted Bidder
s*l(27)**	**0.99981**	**0.99999**	**0.00001**	**0.00019**	**Shill**
6***o(3)	0.74710	0.74868	0.25132	0.25290	Suspect
P***P(8)	0.12798	0.13083	0.86917	0.87202	Trusted Bidder
P***k(0)	0.21782	0.22180	0.77820	0.78218	Trusted Bidder
a***l(20)	0.11713	0.12028	0.87972	0.88287	Trusted Bidder
i***e(7)	0.15599	0.15909	0.84091	0.84401	Trusted Bidder
n***0(8)	0.66078	0.66298	0.33702	0.33922	Suspect
v***i(0)	0.28270	0.28542	0.71458	0.71730	Trusted Bidder

bel(shill) ≥ 0.95 => Shill
0.95 > bel(shill) >0.5 => Suspect
bel(shill) ≤ 0.5 => Trusted Bidder

Fig. 5. Certification assignment rules

6.3. *Analysis & discussion*

6.3.1 Certification result analysis

We now analyze the auction data and the certification results by considering three levels of certification. In the Xbox auction, there are totally 12 bidders and 61 bids in the auction. At the end of the certification process, 8 bidders are certified as *Trusted Bidder*, 3 bidders are certified as *Suspect*, and 1 bidder is certified as *Shill*. We first examine the auction-level evidence. The auction attracted 61 bids. This number is much higher than the average number of bids, which is 7.67, in the same type of auctions. Note that this is not due to the lower price of this auction than those of the concurrent auctions, because the final price of the auction is $167.5, which is higher than the average final price, $138.94, of auctions selling the same item. Furthermore, the average starting price of auctions selling the same product is $40.64, but the starting price of this auction is merely $0.01. While lower starting price may attract bidders to the auction, there is also a higher probability that the seller planed to employ shills to set up a hidden reserve price in order to sell the item at a satisfactory price. The auction-level analysis supports that the auction under investigation may involve shills.

We now investigate various bidders with different certifications to see if our manual investigation is consistent with the certification result.

Shill: The system only certifies one bidder, *s***l* as a shill in this auction. Given Eq. (33) – (37), the degrees of belief from available evidence are combined to obtain the joint belief of shill. The belief of shill for *s***l* is 0.9998, which is greater than φ (0.95), thus the system updates the certification of *s***l* to *Shill*. There are reasons to consider this ultimate result as reasonable because this bidder's behavior is very suspicious. First, *s***l* has a very obvious bidding pattern as shown in Table 7. In most of the Xbox 360 auctions that were hosted by the same seller, *s***l* joined the auction in the middle, placing a proxy bid at $75, and then when outbid, this bidder increases the bid to $100. Most of the time, this bidder stops bidding at $125. From this bidding pattern, it looks like *s***l* is driving up the price and outbidding the potential buyers until the price is relatively high (e.g., $125) and the risk of winning is high. Second, *s***l* has placed bids in 30 out of 36 auctions that were hosted by the same seller. The high number indicates *s***l* and the seller have a strong business relationship. However, the winning ratio for *s***l* is low. Even though he has won 5 auctions, the winning price is relative low. These wins are most likely accidental wins because most of the winning prices are $100 or $125, which looks to be the habitual bid values *of s***l*. So, bidder *s***l* was forced to win the auction when nobody placed a higher bid. Third, in almost all of the auctions, *s***l* has placed bids with increments 40 times the minimum increment. Note that typical normal bidders tend to bid cautiously, with the minimum increment. Fourth, *s***l* placed as high as 70 abnormal concurrent bids in one month. Bidder *s***l* might know that the price in the specific seller's auction was higher than that of a concurrent auction and that this auction would end later than the other one, yet *s***l* still placed bids on the seller's auction.

All in all, the evidence is consistent with the certification results. Interestingly, several days later after we collected the data, we found that *s***l* became labeled as "No Longer A Registered User (NLARU)" in eBay. According to eBay's explanation, NLARU means the bidder's account is suspended by eBay due to violations of eBay's policy, such as shill bidding, selling counterfeit item, keyword spamming, transaction outside eBay. Although we would not be able to know the actual reason why the account was suspended, as a bidder, the most possible reason for *s***l* to be labelled NLARU is due to shill bidding. This once again helps confirm the shill certification result.

Table 7. A suspicious bidding history for s***l

Winner	Winning bid	Num. of Bidders	Num. of bids	Bids of s***l(27)
a*a(4)**	168.49	7	20	80-115
g*o(20)**	202.5	8	18	NA
t*e(2)**	149.5	13	31	NA
s*l(27)**	100	6	14	100
z*u(1)**	147.5	7	15	100-110-115-120-125-140-145
3*6(163)**	171	15	27	70-100-110
d*d(195)**	168.5	8	16	100-120-130
0*1(0)**	152.5	12	32	130-140
e*e(642)**	167.5	12	61	100-116-128

o*i(2)**	129.5	9	23	88-127
6*o(3)**	137.5	6	27	100-120-127-135
o*d(9)**	137.5	15	24	100-125
b*d(27)**	152.5	11	22	100-130-140

Suspect: For suspects, the evidence is not sufficient enough to support the bidders as shills but it is still more sufficient than evidence that supports *~shill*. In the case study, the system gives the certification of *Suspect* to three bidders. They are bidders *o***i*, *6***o*, and *n***0*. We first justify bidder *o***i*. The statistics shows that *o***i* placed bids in 6 out of 36 auctions hosted by the same seller on different dates. He placed many bids in Xbox auctions but won only once at a very low price ($125.5). This behavior matches one of the most significant shilling behavior characteristics: bids frequently, but seldom wins. Besides, *o***i* placed 11 abnormal concurrent bids in the seller's auction. This evidence further enforces the belief that *o***i* is suspicious. However, *o***i* placed his last bid at the final stage of the auction, and also placed bids in many other sellers' auctions. There is strong evidence that supports both sides. Therefore, this bidder is certified as a shill suspect. For the other two bidders, we can observe that the wining ratio (Wins Per Bid) of *6***o* for the specific seller is lower than his average winning ratio, and the increment of *n***0* is exceptionally high. These behaviors make the shill evidence more sufficient than the honest evidence. Therefore, bidder 6***o and bidder *n***0* are certified as suspect at this moment. The certification result is again consistent with our manual investigation.

Trusted Bidder: We show what kind of bidders is considered honest. Without much doubt, *e***e* is not a shill because *e***e* only participated once in the seller's auctions, and won at the end. This win is not accidental since *e***e* placed all of his bids in the final stage of the auction. Now we consider bidders *s***h* and *f***a*. Even though they contribute all of their bids to the seller and they did not win any auction, both of them only participated in only one auction, and they placed bids in the final stage of the auction. Such bidding behavior indicates that they are at least not afraid of winning. Therefore, they are most likely typical cautious new bidders. The reason why *p***p* is not a shill or shill suspect is that *p***p* placed bids in five different sellers' auctions and *p***p* finally won one Xbox game system auction that was hosted by other seller. For the other two bidders, *p***k* and *a***l*, it is easily to see that they placed bids on more than one Xbox system auctions hosted by different sellers. Although they did not win any of the auctions, they did not place any abnormal concurrent bid and they increased their bids cautiously. To sum up, our calculation results are consistent with our manual investigation for identifying trusted bidders.

6.3.2 Discussion

To study the importance of the auction-level evidence, we performed the shill certification again, but using only bid-level evidence. The shill certification results for the same auction data collected from eBay are listed in Table 7. Based on the experimental results, we found that with auction-level evidence, the values for belief-of-shill can be amplified, so some bidders (O***i(2), 6***O(3), and n***0(8)) considered as normal bidders (without auction-level evidence) are now considered as Suspects. This would be valuable to identify any suspicious bidders who can then be subjected to further investigation. Note that a detailed discussion on the impacts of adopting different levels of evidence is beyond the scope of this paper, but it is envisioned as useful for future work.

Table 7 Shill certification results without auction-level evidence

Bidder	bel(shill)	pl(shill)	bel(~shill)	pl(~shill)	Result
e***e(641)	0	8.63722E-05	0.9999	1	Trusted Bidder
O***i(2)	0.0714	0.0899	0.9102	0.9286	Trusted Bidder
s***h(0)	0.0007	0.0010	0.9990	0.9993	Trusted Bidder
f***a(0)	0.0007	0.0010	0.9990	0.9993	Trusted Bidder
s*l(27)**	**0.9972**	**0.9999**	**0.0001**	**0.0028**	**Shill**
6***O(3)	0.1666	0.1718	0.8282	0.8334	Trusted Bidder
P***P(8)	0.0071	0.0104	0.9896	0.9929	Trusted Bidder
P***k(0)	0.0144	0.0195	0.9805	0.9856	Trusted Bidder
a***l(20)	0.0059	0.0094	0.9906	0.9941	Trusted Bidder
i***e(7)	0.0094	0.0130	0.9869	0.9906	Trusted Bidder
n***0(8)	0.1147	0.1205	0.8795	0.8852	Trusted Bidder
v***i(0)	0.0234	0.0271	0.9729	0.9766	Trusted Bidder

7. Related Work

In this section, we review related work on applications of D-S theory, as well as online auction trust management.

Dempster-Shafer Theory. Information related to decision making is often uncertain and incomplete. Hence it is of vital importance to find a feasible way to manage and make decisions under uncertainties. Dempster-Shafer (D-S) theory [9], a probabilistic reasoning technique, is designed to deal with uncertainties and incompleteness of available information. It is a powerful tool for combining accumulative evidence and changing prior knowledge in the presence of new evidence. D-S theory has been used widely for intrusion detection, fraud detection and system verification. For example, Chen, et al., presented a D-S theory based intrusion detection approach for Ad Hoc networks [17]. In their system, data from multiple processors are combined to form a decision about a node's true identity. Panigrahi, et al., used D-S theory to combine evidence to estimate the likelihood of fraud in the context of mobile phone fraud detection [18]. They demonstrated the effectiveness of D-S theory for their applications.

In this paper, we propose a unique shill verification method based on D-S theory. Our proposed methods can be used to combine multiple pieces of evidence with new evidence to categorize a shill suspect's genuineness, and thereafter to update the suspect's status in a complex shill detection system.

Trust management in online auction systems. Xu, et al., presented an Agent-based Trust Management (ATM) framework for online auctions [19]. The shill certification procedure discussed in this paper can be embedded in the security agent of ATM. Xu, et al., also introduced a formal model checking approach to detecting shilling behaviors, especially the competitive shilling behaviors [6]. Kauffman, et al., statistically analyzed data from rare coin auctions on eBay, and empirically tested the questionable bidding behaviors that are attributable to shill bidding [5]. Rubin, et al., proposed a new reputation system that especially indicates the likelihood of shilling behaviors for auction sites [20]. Trevathan, et al., designed an algorithm based on pattern matching to detect shilling behaviors in online English auctions [21]. Shah, et al., mined associations between buyers and sellers and found one indicator of shill bidding: bidders only bid in auctions hosted by one particular seller and seldom won [22]. Chau, et al., applied data mining and trust propagation techniques to detect fraudulent users in online auction systems [23]. Bhargava, et al., derived an equilibrium bidding strategy called shill counteracting bidding strategy (SCBS) to help honest bidders counteract shills in English auctions. Both theoretical and experimental results confirm that the equilibrium bidding strategy increases the bidders' expected utility [24, 25]. More recently, Xu, et al., proposed a formal approach to detecting shill bidders in live online auctions [15]. The approach introduced a dynamic auction model (DAM), and used real-time model checking techniques to verify shilling behaviors specified using linear temporal logic (LTL).

Generally, most of the existing techniques suffer from two drawbacks. Data mining related approaches need to deal with a large amount of historical data; thus they may have limited value in detecting shill bidding in a time-efficient manner. Pattern matching based approaches do not regularly update prior knowledge with the presence of new evidence, i.e., these techniques do not update prior findings every time a new piece of evidence is observed. Therefore they may frequently generate false positive results. In contrast, our proposed approach can not only detect suspicious shilling behavior timely, but can also make the results more accurate and helpful for online auctions. In this sense, our approach complements existing approaches such as the real-time model checking approach [15] that requires analyzing real-time auction data, which is very efficient but may lead to inaccurate results.

8. Conclusions and Future Work

Based on the conceptual framework of Dempster-Shafer theory, a unique practical shill detection approach has been proposed. This method in essence takes evidence from different levels, i.e., auction-level and bid-level, into consideration. The knowledge from auction properties and bidding behaviors are represented and quantified. Using Dempster's rule of combination, we combined evidence that enforces each other and resolved the conflicts between different pieces of evidence. The case study shows that our proposed approach is accurate and practical for real world deployment.

In future research, we plan to design a shill detection agent with self learning capability so that the parameters used in our approach can be optimized automatically. We also plan to consider user profiles to assist the process of shill detection. We believe that the Dempster-Shafer theory, as a theoretically generalized Bayesian inference method, can provide a practical approach and enhance system performance for shill detection in online auctions.

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